

BELLCOMM, INC.

955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D. C. 20024

SUBJECT: Forces on a Current-Carrying  
Coil in the Earth's Magnetic  
Field - Case 620

DATE: February 14, 1969

FROM: R. J. Ravera

ABSTRACT

There have been some recent studies to determine if current carrying coils might be used as torquers for attitude control of the AAP Cluster by reacting with the earth's magnetic field. A side aspect of this problem is that forces act on these coils due to the gradient of the field. The question is whether these forces will have an appreciable effect on orbital parameters. A general expression for the force on a coil in a magnetic field is derived. A dipole model of the earth's magnetic field and the properties of a coil suitable for attitude control are used to obtain numerical results for 210 NM circular orbits in the equatorial plane and inclined at 35 degrees. Three orientations of the coil are studied; Case (a) assumes the plane of the coil in the plane of the orbit and in Cases (b) and (c) the plane of the coil is perpendicular to the orbital plane with its axis inertially fixed. In Case (b) the normal to the coil area is in the direction of the line of nodes and in Case (c) it is normal to the line of nodes.

The maximum radial component of the force occurs in Case (a) for the 35° inclination. Its value is approximately  $3.5 \times 10^{-6}$  lbs, which is about  $3.5 \times 10^{-11}$  of the gravitational pull on the cluster. The maximum tangential ("drag") component of the force occurs in Cases (b) and (c) for the 35° inclination. Its magnitude is approximately  $10^{-6}$  lbs which is in the range  $10^{-3}$ - $10^{-4}$  of the aerodynamic drag on the cluster. It should be noted that the tangential component of the force first boosts and then opposes the motion of the vehicle in alternate quarters of the orbit. The maximum value of the component of force in the direction perpendicular to the orbital plane is about  $1.6 \times 10^{-6}$  lbs, and points first southward and then northward in alternate halves of the orbit.

The force on a current carrying coil is negligible compared to both gravitational and drag forces. Note that this conclusion refers to the force and not the torque generated by the field. The force effect should not, therefore, detract from the magnetic control concept, but neither can it be used to advantage for orbit keeping.

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(NASA-CR-104011) FORCES ON A  
CURRENT-CARRYING COIL IN THE EARTH'S  
MAGNETIC FIELD (Bellcomm, Inc.) 16 p

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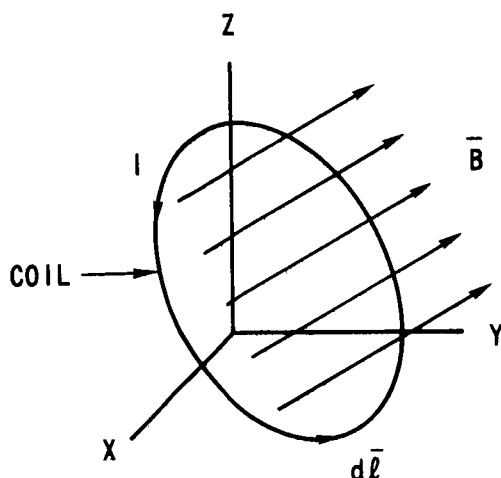
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MEMORANDUM FOR FILEI. INTRODUCTION

There have been a number of recent studies to determine the feasibility of using current carrying coils as torquers for attitude control of the AAP cluster by reacting with the earth's magnetic field. Due to the gradient existing in the field, forces as well as torques act on the coils. The object of this memorandum is to study these forces first from a general point of view, and then to determine if their magnitudes will have an appreciable effect on orbital parameters. The numerical results are based on a coil suitable for attitude control in a 210 NM circular orbit with inclinations of  $0^\circ$  and  $35^\circ$ . Three coil orientations with respect to the orbital plane are considered; the coil contained in the orbital plane and two distinct orientations in which the coil is perpendicular to the orbital plane.

II. FORCE ON A COIL IN A MAGNETIC FIELD

Consider a coil carrying a current  $I$  which is placed in a magnetic field  $\vec{B}$ . The force acting on an element of the coil,  $d\vec{l}$ , is

$$d\vec{F} = I [d\vec{l} \times \vec{B}]$$

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One can compute the total resultant force on the coil by evaluating some line integrals; from (1),

$$\bar{F} = \oint_C d\bar{F} = I \oint_C [d\bar{\ell} \times \bar{B}] \quad (2)$$

However, rather than evaluate (2) through the tedious process of carrying out line integrals we can use one of the Stoke's related formulas from general vector analysis.\* The resultant force from (2) becomes

$$\bar{F} = \oint_C d\bar{F} = I \oint_C [d\bar{\ell} \times \bar{B}] = I \int_S (d\bar{A} \times \nabla) \times \bar{B}. \quad (3)$$

where  $d\bar{A} = \bar{n} dA$ , with  $dA$  being an element of area enclosed by the coil and  $\bar{n}$  the unit normal to the element of area positive in the right-hand sense when traversing the contour so that the area is always to the left. Then (3) becomes

$$\bar{F} = I \int_S [(\bar{n} \times \nabla) \times \bar{B}] dA \quad (4)$$

Expanding equation (4) provides some insight. If  $\bar{i}$ ,  $\bar{j}$ , and  $\bar{k}$  are unit vectors in the x, y, and z directions, then

$$\bar{n} = n_x \bar{i} + n_y \bar{j} + n_z \bar{k} \quad (5)$$

$$\bar{B} = B_x \bar{i} + B_y \bar{j} + B_z \bar{k} \quad (6)$$

$$\text{and} \quad \nabla = \frac{\partial}{\partial x} \bar{i} + \frac{\partial}{\partial y} \bar{j} + \frac{\partial}{\partial z} \bar{k} \quad (7)$$

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\*See Simons, S., Vector Mechanics, P. 139.

The resultant force on the coil is, after expansion

$$\begin{aligned} \bar{F} = I \int_S \left\{ \left[ n_z \frac{\partial B_z}{\partial x} + n_y \frac{\partial B_y}{\partial x} - n_x \left( \frac{\partial B_z}{\partial z} + \frac{\partial B_y}{\partial y} \right) \right] \bar{i} \right. \\ + \left[ n_x \frac{\partial B_x}{\partial y} + n_z \frac{\partial B_z}{\partial y} - n_y \left( \frac{\partial B_x}{\partial x} + \frac{\partial B_z}{\partial z} \right) \right] \bar{j} \\ \left. + \left[ n_x \frac{\partial B_x}{\partial z} + n_y \frac{\partial B_y}{\partial z} - n_z \left( \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} \right) \right] \bar{k} \right\} dA \end{aligned} \quad (8)$$

Observe that if  $\bar{B}$  lies completely in one plane, say the XY-plane, and the coil also lies completely in a plane parallel to the XY-plane, then

$$B_z = 0$$

and

$$\bar{n} = \bar{k}$$

or

$$n_x = n_y = 0, n_z = 1$$

and Equation (8) becomes

$$\begin{aligned} \bar{F} &= [-I \int_S \left( \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} \right) dA] \bar{k} \\ &= [-I \int_S (\nabla \cdot \bar{B}) dA] \bar{k} \end{aligned} \quad (9)$$

Equation (9) is a particular case result while Equation (8) is completely general.

II. SOLENOIDAL FIELD

A vector field  $\vec{F}$  is said to be solenoidal if  $\nabla \cdot \vec{F} = \text{Div } \vec{F} = 0$ . The magnetic induction field,  $\vec{B}$ , is always solenoidal.\* Thus,

$$\nabla \cdot \vec{B} = \text{Div } \vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 \quad (10)$$

A direct consequence of (10) is that if the magnetic field  $\vec{B}$  and the coil are both contained completely in the same plane, it follows from (9) that

$$\vec{F} = 0. \quad (11)$$

Equation (10) also alters the more general result of (8). From (10),

$$\begin{aligned} \frac{\partial B_x}{\partial x} &= - \left( \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} \right) \\ \frac{\partial B_y}{\partial y} &= - \left( \frac{\partial B_x}{\partial x} + \frac{\partial B_z}{\partial z} \right) \\ \frac{\partial B_z}{\partial z} &= - \left( \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} \right) \end{aligned} \quad (12)$$

Combining (12) and (8) leads to

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\* See Abraham, M. and Becker, R., Electricity and Magnetism, P. 129.  $\text{Div } \vec{B}=0$  means physically that there exist no free "magnetic sources" or "charges".

$$\begin{aligned} \bar{F} = I \int_S \left\{ \left[ n_x \frac{\partial B_x}{\partial x} + n_y \frac{\partial B_y}{\partial x} + n_z \frac{\partial B_z}{\partial x} \right] \bar{i} \right. \\ + \left[ n_x \frac{\partial B_x}{\partial y} + n_y \frac{\partial B_y}{\partial y} + n_z \frac{\partial B_z}{\partial y} \right] \bar{j} \\ \left. + \left[ n_x \frac{\partial B_x}{\partial z} + n_y \frac{\partial B_y}{\partial z} + n_z \frac{\partial B_z}{\partial z} \right] \bar{k} \right\} dA \end{aligned}$$

or in vector short-hand,

$$\bar{F} = I \int_S \left\{ [\bar{n} \cdot \frac{\partial \bar{B}}{\partial x}] \bar{i} + [\bar{n} \cdot \frac{\partial \bar{B}}{\partial y}] \bar{j} + [\bar{n} \cdot \frac{\partial \bar{B}}{\partial z}] \bar{k} \right\} dA \quad (13)$$

Equation (13) is valid for any coil in a solenoidal field ( $\nabla \cdot \bar{B} = 0$ ). Equation (11) is valid for a particular orientation of the coil only.

### III. FURTHER SIMPLIFICATION

Suppose now that the coil is contained completely in any plane. This is equivalent to specifying a constant  $\bar{n}$  over the entire area bounded by the coil. Then (13) becomes

$$\begin{aligned} \bar{F} &= I \int_S \left\{ \left[ \frac{\partial}{\partial x} (\bar{n} \cdot \bar{B}) \right] \bar{i} + \left[ \frac{\partial}{\partial y} (\bar{n} \cdot \bar{B}) \right] \bar{j} + \left[ \frac{\partial}{\partial z} (\bar{n} \cdot \bar{B}) \right] \bar{k} \right\} dA \\ &= I \int_S [\text{grad } (\bar{n} \cdot \bar{B})] dA \\ &= I \int_S [\text{grad } B_n] dA \end{aligned} \quad (14)$$

where  $B_n$  is the component of  $\bar{B}$  along the normal to the area bounded by the coil and

$$\text{grad } B_n = \frac{\partial B_n}{\partial x} \bar{i} + \frac{\partial B_n}{\partial y} \bar{j} + \frac{\partial B_n}{\partial z} \bar{k}$$

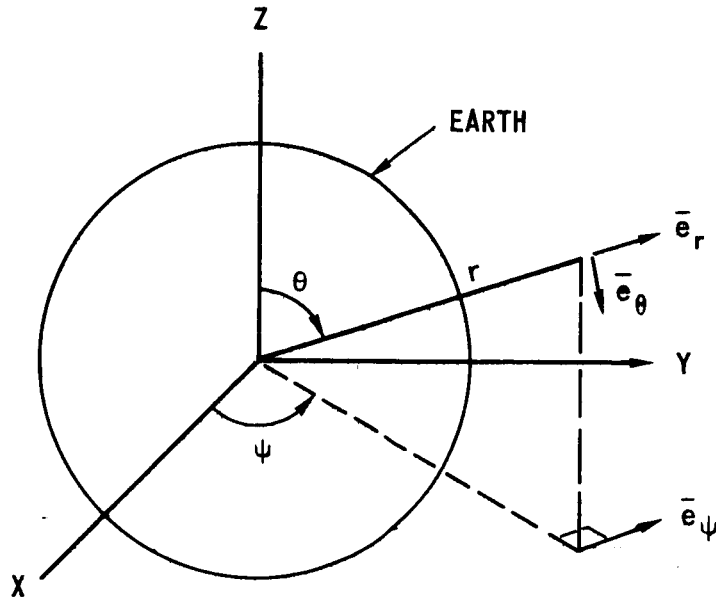
IV. EXAMPLE OF COIL IN EARTH'S MAGNETIC FIELD

FIGURE 1

The earth's magnetic field will be represented by a magnetic dipole whose dipole vector is aligned with the earth's spin vector. The north pole lies on the z axis in Figure 1. In spherical polar coordinates, the field is given by

$$\vec{B} = -\frac{M_e}{r^3} [2 \cos \theta \vec{e}_r + \sin \theta \vec{e}_\theta]^* \quad (15)$$

Where  $M_e$  is the earth dipole moment. The coordinates  $r$ ,  $\theta$ , and  $\psi$  and the unit vectors  $\vec{e}_r$ ,  $\vec{e}_\theta$  and  $\vec{e}_\psi$  are defined in Figure 1. Our primary interest is to determine the radial and tangential (drag) components of the force on a current carrying coil which is in an orbit inclined to the equator through an angle  $i$ , the x-axis being the line of nodes. It will then prove convenient to describe  $B_n$  in the spherical coordinate system shown in Figure (2), obtained by

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\*It is easily verified that  $\nabla \cdot \vec{B} = \text{Div } \vec{B} = 0$

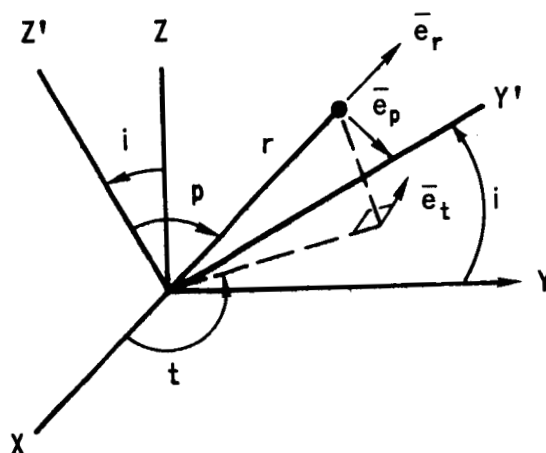


FIGURE 2

rotating the z and y-axes through an angle  $i$  about the x-axis. When  $p = \pi/2$ ,  $r$  will be in the desired orbital ( $xy'$ -) plane while  $\bar{e}_r$ ,  $\bar{e}_p$  and  $\bar{e}_t$  will represent unit vectors which are radial, perpendicular to the orbital plane (POP) and tangent to the orbit, respectively. The relationships between  $r\theta\psi$ -coordinates and  $r\theta\psi$ -coordinates are

$$\left. \begin{aligned} \sin \theta \cos \psi &= \sin p \cos t \\ \sin \theta \sin \psi &= \sin p \sin t \cos i - \cos p \sin i \\ \cos \theta &= \sin p \sin t \sin i + \cos p \cos i \end{aligned} \right\} (16)$$

$$r = r$$

Three distinct orientations of the coil in the orbital plane will be considered. The orientations will be denoted by Case (a), Case (b) and Case (c).

a. Case (a) - Coil in orbital plane

If the coil is contained completely in the orbital plane with its magnetic dipole pointing northward then the normal to its area is

$$\bar{n} = -\sin i \bar{j} + \cos i \bar{k}$$



or, in  $r\theta\psi$ -coordinates,

$$\begin{aligned}\bar{n} = & (-\sin i \sin \theta \sin \psi + \cos i \cos \theta) \bar{e}_r \\ & - (\sin i \cos \theta \sin \psi + \cos i \sin \theta) \bar{e}_\theta \\ & - \sin i \cos \psi \bar{e}_\psi\end{aligned}\quad (17)$$

From (15) and (17)

$$\begin{aligned}B_n &= \bar{B} \cdot \bar{n} \\ &= \frac{M_e}{r^3} [2\cos\theta (\sin i \sin\theta \sin \psi - \cos i \cos\theta) \\ &\quad + \sin\theta (\sin i \cos\theta \sin\psi + \cos i \sin\theta)] \\ &= \frac{M_e}{r^3} [3 \sin i \cos\theta \sin\theta \sin\psi + \cos i (1-3 \cos^2\theta)]\end{aligned}\quad (18)$$

In order to obtain  $B_n$  in terms of  $rpt$ -coordinates, combine (16) and (18) to obtain

$$\begin{aligned}B_n = \frac{M_e}{r^3} \left\{ \frac{3}{2} \sin i [\sin^2 i \sin^2 p \sin^2 t + \sin^2 p \sin t (\cos^2 i - \sin^2 i) \right. \\ \left. - \sin 2i \cos^2 p] + \cos i [1-3(\sin^2 i \sin^2 p \sin^2 t \right. \\ \left. + \frac{1}{2} \sin 2i \sin 2p \sin t + \cos^2 i \cos^2 p)] \right\}\end{aligned}\quad (19)$$

Considering variations in  $r$ ,  $p$ , and  $t$  negligible over the region of integration, (14) may be approximated to a high degree of accuracy by

$$\bar{F} \approx IA(\text{grad } B_n)_{r=r_c}\quad (20)$$

where  $A$  is the cross sectional area enclosed by the coil and  $r_c$  is the distance from the center of the earth to the center of the coil.

In spherical polar coordinates

$$\text{grad} ( ) = \frac{\partial}{\partial r} ( ) \bar{e}_r + \frac{1}{r} \frac{\partial}{\partial p} ( ) \bar{e}_p + \frac{1}{r \sin p} \frac{\partial}{\partial t} ( ) \bar{e}_t$$

and from (19)

$$\begin{aligned} \text{grad} (B_n) = & \\ -3 \frac{M_e}{r^4} & \left\{ \frac{3}{2} \sin i [\sin 2i \sin^2 p \sin^2 t + \sin 2p \sin t (\cos^2 i - \sin^2 i)] \right. \\ & - \sin 2i \cos^2 p] + \cos i [1 - 3(\sin^2 i \sin^2 p \sin^2 t \\ & + \frac{1}{2} \sin 2i \sin 2p \sin t + \cos^2 i \cos^2 p)] \left. \right\} \bar{e}_r \\ + \frac{M_e}{r^4} & \left\{ 3 \sin i [\frac{1}{2} \sin 2i \sin 2p (1 + \sin^2 t) + \cos p \sin t (\cos^2 i - \sin^2 i)] \right. \\ & - 3 \cos i [\sin^2 i \sin 2p \sin^2 t + \sin 2i \cos 2p \sin t - \cos^2 i \sin 2p] \left. \right\} \bar{e}_p \\ + \frac{M_e}{r^4 \sin p} & \left\{ \frac{3}{2} \sin i [\sin 2i \sin^2 p \sin 2t + \sin 2p \cos t (\cos^2 i - \sin^2 i)] \right. \\ & - 3 \cos i [\sin^2 i \sin^2 p \sin 2t + \frac{1}{2} \sin 2i \sin 2p \cos t] \left. \right\} \bar{e}_t \quad (21) \end{aligned}$$

It is now noted that the  $z'$  axis in Figure 2 is normal to the orbital plane; therefore angle  $p = \pi/2$ . By combining (20) and (21), setting  $p = \pi/2$  and writing the resultant force in terms of  $F_r$ ,  $F_p$  and  $F_t$  components, we have

$$F_r = -3 \frac{M_e I A}{r_c^4} \left\{ \cos i + 3[\sin i \sin 2i - 3 \sin^2 i] \sin^2 t \right\} \quad (22)$$

$$F_p = \frac{M_e I A}{r_c^4} \left\{ 3 \cos i \sin 2i \sin t \right\}, \quad (23)$$

$$F_t = 0 \quad (24)$$

Note that if the inclination angle  $i=0$  (orbital plane is the equatorial plane)

$$F_r = -3 \frac{M_e I A}{r_c^4} \quad (25)$$

$$F_p = 0 \quad (26)$$

$$F_t = 0 \quad (27)$$

b. Case (b) - Coil perpendicular to orbital plane with normal in the direction of line of nodes

If the coil is oriented perpendicular to the orbital plane so that the normal to the area is always pointing in the direction of the line of nodes,

$$\bar{n} = \bar{i} = \sin \theta \cos \psi \bar{e}_r + \cos \theta \cos \psi \bar{e}_\theta - \sin \psi \bar{e}_\psi \quad (28)$$

From (15) and (28) it follows

$$B_n = \bar{B} \cdot \bar{n} = -3 \frac{M_e}{r^3} [\cos \theta \sin \theta \cos \psi] \quad (29)$$

Combining (16) and (29) leads to

$$B_n = -3 \frac{M_e}{2r^3} [\sin i \sin^2 p \sin 2t + \cos i \sin 2p \cos t] \quad (30)$$

The gradient of  $B_n$  is therefore

$$\begin{aligned}
\text{grad } B_n &= 9 \frac{M_e}{2r^4} [\sin i \sin^2 p \sin 2t + \cos i \sin 2p \cos t] \bar{e}_r \\
&- 3 \frac{M_e}{r^4} [\sin i \sin 2p \sin 2t + \cos i \cos 2p \cos t] \bar{e}_p \\
&- 3 \frac{M_e}{r^4 \sin p} [\sin i \sin^2 p \cos 2t - \frac{1}{2} \cos i \sin 2p \sin t] \bar{e}_t \quad (31)
\end{aligned}$$

Equations (20) and (31) give for the components of the force on the coil (with  $p=\pi/2$ )

$$F_r = 9 \frac{M_e}{2r_c^4} IA \sin i \sin 2t \quad (32)$$

$$F_p = 3 \frac{M_e}{r_c^4} IA \cos i \cos t \quad (33)$$

$$\text{and } F_t = -3 \frac{M_e}{r_c^4} IA \sin i \cos 2t \quad (34)$$

Note that for an equatorial orbit ( $i=0$ )

$$F_r = 0 \quad (35)$$

$$F_p = 3 \frac{M_e}{r_c^4} IA \cos t \quad (36)$$

$$F_t = 0 \quad (37)$$

- c. Case (c) - Coil perpendicular to orbital plane with normal perpendicular to the line of nodes

Here, the normal to the coil area is

$$\begin{aligned}
 \bar{n} &= \cos i \bar{j} + \sin i \bar{k} \\
 &= (\cos i \sin \theta \sin \psi + \sin i \cos \theta) \bar{e}_r \\
 &+ (\cos i \cos \theta \sin \psi - \sin i \sin \theta) \bar{e}_\theta \\
 &+ \cos i \cos \psi \bar{e}_\psi
 \end{aligned} \tag{38}$$

It can be shown that  $B_n$  in terms of rpt - coordinates is

$$\begin{aligned}
 B_n = -\frac{M_e}{r^3} \left\{ 3 \cos i \left[ \frac{1}{2} \sin 2i \sin^2 p \sin^2 t + \frac{1}{2} \sin 2p \sin t (\cos^2 i - \sin^2 i) \right. \right. \\
 \left. \left. - \frac{1}{2} \sin 2i \cos^2 p \right] - \sin i \left[ 1 - 3(\sin^2 i \sin^2 p \sin^2 t \right. \right. \\
 \left. \left. + \frac{1}{2} \sin 2i \sin 2p \sin t + \cos^2 i \cos^2 p \right] \right\}.
 \end{aligned} \tag{39}$$

Taking the gradient of (39), combining it with (20) and setting  $p = \pi/2$  gives for the resultant components of the force

$$F_r = 3 \frac{M_e IA}{r_c^4} \sin i [3 \sin^2 t - 1] \tag{40}$$

$$F_p = -3 \frac{M_e IA}{r_c^4} [(\sin^2 i \cos i - \cos^3 i) \sin t] \tag{41}$$

$$F_t = -3 \frac{M_e IA}{r_c^4} \sin i \sin 2t \tag{42}$$

For an equatorial orbit ( $i=0$ )

$$F_r = 0 \quad (43)$$

$$F_p = 3 \frac{M_e I A}{r_c^4} \sin t \quad (44)$$

$$F_t = 0 \quad (45)$$

## V. NUMERICAL RESULTS

Preliminary results\* indicate that a coil made of aluminum cable, circular in shape with a 100 ft diameter and weighing about 195 lbs is an appropriate design on which to base the numerical results. The average power is 411.5 watts and current (RMS) is 222.8 amps.

The coil will be assumed to be in a circular 210 NM orbit\*\* and inclinations of  $0^\circ$  and  $35^\circ$  will be considered.

### Case (a)

$i=0^\circ$  (From (25), (26) and (27))

$$\begin{cases} F_r = -1.62 \times 10^{-6} \text{ lbs.} \\ F_p = F_t = 0 \end{cases}$$

$i=35^\circ$  (From (22), (23) and (24))

$$\begin{cases} F_r = -1.62 \times 10^{-6} [0.81915 - 1.34 \sin 2t] \text{ lbs.} \\ F_{r_{\max}} = -3.49 \times 10^{-6} \text{ lbs @ } t = \frac{3\pi}{4}, \frac{7\pi}{4} \\ F_p = 1.25 \times 10^{-6} \sin t \text{ lbs.} \\ F_t = 0 \end{cases}$$

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\* Based on discussions with J. Kranton and B. W. Moss.

\*\* Properties of the earth's magnetic field were obtained from the TRW Space Data Handbook, P. 78.

Case (b) $i=0^\circ$  (From (35), (36) and (37))

$$\begin{cases} F_r = F_t = 0 \\ F_p = 1.62 \times 10^{-6} \cos t \text{ lbs.} \end{cases}$$

 $i=35^\circ$  (From (32), (33) and (34))

$$\begin{cases} F_r = 1.34 \times 10^{-6} \sin 2t \text{ lbs.} \\ F_p = 1.33 \times 10^{-6} \cos t \text{ lbs.} \\ F_t = 0.93 \times 10^{-6} \cos 2t \text{ lbs.} \end{cases}$$

Case (c) $i=0^\circ$  (From (43), (44) and (45))

$$\begin{cases} F_r = F_t = 0 \\ F_p = 1.62 \times 10^{-6} \sin t \text{ lbs.} \end{cases}$$

 $i=35^\circ$  (From (40), (41) and (42))

$$\begin{cases} F_r = 0.93 \times 10^{-6} [3 \sin^2 t - 1] \text{ lbs.} \\ F_{r_{\max}} = 1.86 \times 10^{-6} \text{ lbs @ } t = \pi/2, 3\pi/2 \\ F_p = 0.45 \times 10^{-6} \sin t \text{ lbs.} \\ F_t = -0.93 \times 10^{-6} \sin 2t \text{ lbs.} \end{cases}$$

VI. DISCUSSION OF RESULTS

Case (a): With zero orbit inclination, there is only a radial component of the force. It is constant in magnitude and direction, pointing always towards the earth's center.\* At a  $35^\circ$  inclination, the radial component,  $F_r$ , oscillates about a mean value of  $-1.33 \times 10^{-6}$  lbs. and depends on the sine of twice the angle  $t$ . It reaches a maximum value of  $3.49 \times 10^{-6}$  lbs. at  $t = 3\pi/4, 7\pi/4$ . The POP component varies as  $\sin t$ , therefore pointing southward for  $0 < t < \pi$  and northward for  $\pi < t < 2\pi$ . The tangential (drag) component,  $F_t$ , remains zero for the inclined orbit.

Case (b): For  $i=0$ ,  $F_r$  and  $F_t$  are zero while the POP component varies as  $\cos t$  pointing south for  $t$  in the right half plane and north in the left half plane. When  $i=35^\circ$ , the  $F_r$  component depends on  $\sin 2t$  and therefore points away from the earth when  $0 < t < \pi/2, \pi < t < 3\pi/2$  and is toward the earth when  $\pi/2 < t < \pi, 3\pi/2 < t < 2\pi$ . The POP component again oscillates in magnitude and direction according to  $\cos t$ . A tangential or "drag" component,  $F_t$ , makes its first appearance and is approximately equal to  $10^{-6} \cos 2t$  lbs.  $F_t$  switches from a "thrust" force to a "drag" force in alternate quarters of the orbit.

Case (c): The only uniquely different occurrence in this case concerns the dependence of  $F_r$  on  $\sin^2 t$  for  $i=35^\circ$ . The maximum value of  $F_r$  is attained at  $t = \pi/2, 3\pi/2$  and is equal to  $1.86 \times 10^{-6}$  lbs.

Comparing the magnitude of the forces acting on a current carrying coil to other environmental forces, it is found that the ratio of the maximum tangential force to the aerodynamic drag on the cluster is in the range  $10^{-3} - 10^{-4}$ . In addition, the force is of the same order of magnitude as the force due to solar pressure.

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\*Directional properties are naturally dependent on direction of current through the coil.



It can thus be concluded that the force on a current carrying coil is negligible compared to the drag forces anticipated for the AAP mission. The force effect should not, therefore, detract from the magnetic attitude control concept, but neither can it be used to advantage for orbit keeping. However, for other than AAP missions with a comparable size vehicle, there exists a cross-over point in the vicinity of 900 n.m. at which the forces on a current carrying coil can exceed the aerodynamic force. On the other hand, the solar pressure at 900 n.m., which remains relatively constant (being inversely proportional to the square of the distance from the spacecraft to the sun), can give rise to forces in the range of ten times the aerodynamic and coil forces. Therefore, there can exist conditions for which forces in current carrying coils cannot be considered negligible compared to other environmental effects; at the same time, without major changes in the coil design (increases in power and diameter for example), they will probably not constitute the dominant effect.

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Coil in the Earth's Magnetic  
Field - Case 620

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